



ENSEMBLE: MICROCANONICAL

CANONICAL

GRAND CANONICAL

CONSTRAINTS: ISOLATED

ENERGY  
EXCHANGE

ENERGY AND PARTICLE  
EXCHANGE

SPECIFY: U, V, N

$\tau, V, N$

$\tau, V, \mu$

CALCULATE:  $g(U, V, N)$  (from quantum mechanics)  
 $\sigma = \log(g)$   
 $1/\tau = (\partial\sigma/\partial U)_{V, N}$

$Z = \sum_s e^{-\epsilon_s/\tau}$   
 $F = -\tau \log Z$   
 $\sigma = -(\partial F/\partial \tau)_{V, N}$   $P = -(\partial F/\partial V)_{\tau, N}$   
 $U = F + \tau\sigma$

$Z = \sum_N e^{\left(\frac{N\mu}{\tau}\right)} \sum_s e^{\left(\frac{-\epsilon_s(N)}{\tau}\right)}$   
 $\Omega = F - \mu N = -\tau \log Z$   
 $\sigma = -(\partial\Omega/\partial\tau)_{V, \mu}$   $P = -(\partial\Omega/\partial V)_{\tau, N}$   
 $N = -(\partial\Omega/\partial\mu)_{V, \tau}$   
 $F = \Omega + \mu N$   
 $U = \Omega + \mu N + \tau\sigma$

PROBABILITY:  $P_s = 1/g$

$P_s = \frac{e^{-\epsilon_s/\tau}}{Z}$

$P_s = \frac{e^{\left(\frac{N\mu - \epsilon_s(N)}{\tau}\right)}}{Z}$

APPLICATIONS: Fundamentals, deriving other ensembles, physical insight, two-level systems.

Systems where Z can be evaluated: two or multi-level, harmonic oscillator, one particle in a box, photons,...

Treating an *individual* quantum state (orbital) as an open system; calculate  $f = \langle N \rangle$  for each state, where

$$f(\epsilon) = \frac{1}{e^{(\epsilon - \mu)/\tau} \pm 1}$$

then summing over all states to get total N. Classical, Bose (-), and Fermi (+) gases...